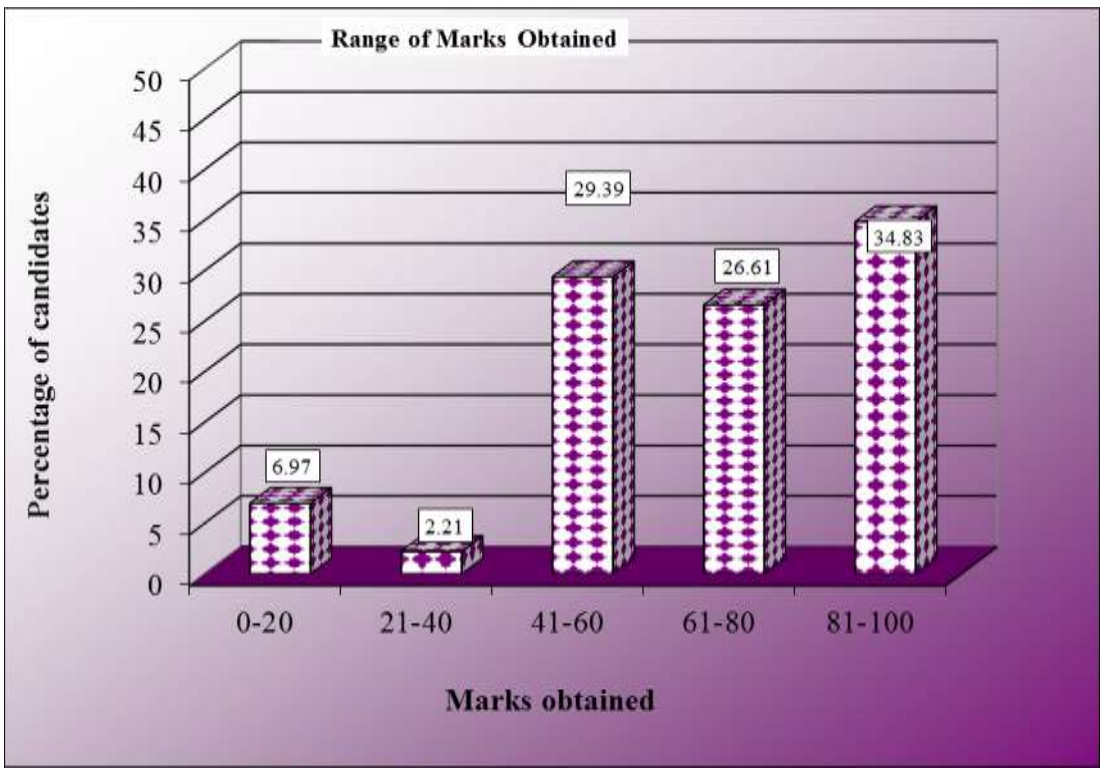
MATHEMATICS

A. STATISTICS AT A GLANCE

Total number of students taking the examination	43,037
Highest marks obtained	100
Lowest marks obtained	1
Mean marks obtained	67.84

Percentage of candidates according to marks obtained

	Mark Range				
	0-20	***21-40***	***41-60***	***61-80***	***81-100***
Number of candidates	2999	949	12648	11452	14989
Percentage of candidates	6.97	2.21	29.39	26.61	34.83
Cumulative Number	2999	3948	16596	28048	43037
Cumulative Percentage	6.97	9.17	38.56	65.17	100



B. ANALYSIS OF PERFORMANCE

SECTION A

Question 1 **[10×3]**

(i) If A = $\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, find the values of x and y such that $A^2 + x I_2 = yA$.

(ii) Find the eccentricity and the coordinates of foci of the hyperbola $25x^2 – 9y^2 = 225$.

(iii) Evaluate: $\tan\left[2\tan^{-1}\frac{1}{2} - \cot^{-1} 3\right]$

(iv) Using L'Hospital's Rule, evaluate:

$\lim_{x \to 0}(1+\sin x)^{\cot x}$

(v) Evaluate: $\int e^x \frac{(2+\sin 2x)}{\cos^2 x} dx$

(vi) Using properties of definite integrals, evaluate: $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$.

(vii) For the given lines of regression, $3x – 2y = 5$ and $x – 4y = 7$, find:

(i) regression coefficients b_{yx} and b_{xy}

(ii) coefficient of correlation $r(x, y)$

(viii) Express the complex number $\frac{\left(1+\sqrt{3}i\right)^2}{\sqrt{3}-i}$ in the form of a + ib. Hence, find the modulus and argument of the complex number.

(ix) A bag contains 20 balls numbered from 1 to 20. One ball is drawn at random from the bag. What is the probability that the ball drawn is marked with a number which is multiple of 3 or 4?

(x) Solve the differential equation:

$$(x + 1)dy – 2\, xy\, dx = 0$$

Comments of Examiners

(i) Some candidates found A^2 by squaring the elements of A, instead of multiplying A by A. I_2 was taken as $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ by a few instead of $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

(ii) For the given Hyperbola $\frac{x^2}{9} - \frac{y^2}{25} = 1$, some mistakenly took the foci on the conjugate (i.e. y) axis instead of on the Transverse axis (i.e. x-axis), since they found the larger number 25 under y^2. Some gave only one focus (ae, 0) instead of two (± ae, 0).

(iii) Some candidates could not convert $2\tan^{-1}\frac{1}{2}$ into $\tan^{-1}\frac{4}{3}$. Some got the formula for $\tan^{-1}x - \tan^{-1}y$ wrong. Several candidates converted into $\sin^{-1}$ or $\cos^{-1}$ forms and got into more difficulties.

(iv) Conversion into $\frac{0}{0}$ (indeterminate) form for the given limit directly or after taking logs on both sides was not done by some candidates. Some differentiated the fraction by $\frac{u}{v}$ rule instead of separately differentiating the numerator and denominator as is required in L' Hospitals Rule.

(v) Several candidates could not convert the given integral, $\int e^x.F(x)\,dx$ into two parts of the form $\int e^x \{f(x) + f'(x)\}\,dx$ and then use by parts rule or formula.

(vi) Use of the definite integral property $\int_o^a f(x)dx = \int_o^a f(a-x)dx$ was tired but unsuccessfully by some candidates. Some tried direct integration or integration by substitution and failed after a point.

(vii) The concept of regression coefficient (b_{yx} and b_{xy}) was not clear to a number of candidates. Many candidates solved the given equations for x and y unnecessarily. Some forget that the product of the regression coefficients yields a position fraction less than unity.

Suggestions for teachers

- All terms need to be defined well and operations taught and explained in a manner that students understand.
- Help students avoid confusion between ellipse and hyperbola. The relative values of a^2 and b^2 for the Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ does not determine CA and TA but rather the position of +ve and –ve signs.
- A revision of basis trigonometry laws is a must. All concepts of inverse Trigonometry functions must be understood by the students including conversation, combination, double and triple angle laws through sufficient practice.
- L' Hospitals rule and when and how it can be applied needs to be demonstrated in a way that the students understand. Indeterminate forms $\frac{0}{0}$, $\frac{00}{00}$.etc. need to be explained.
- $\int e^x \{f(x) + f'(x)\}\,dx$ is specific situation in Integration By Parts. This needs to be sufficiently practiced by students. Also, make students revise trigonometric and algebraic laws as well as basic integrals.
- Properties of definite integral need to be taught properly and the effective use of a property needs to be understood by students. Teach that properties are used for simplifying the integral into easier forms.

(viii) A number of candidates failed to convert the given function into the correct a + ib form. Hence the modulus and argument were incorrect. Some took $\left(\sqrt{3}i\right)^2$ as $9i^2$ instead of $3i^2$. Some did not know that the value of argument depends on the quadrant in which the complex number lies.

(ix) Many candidates did not correctly obtain the cases in favour of the number that were multiples of 3 or 4. Some used the formula $P(A\cup B) = P(A) + P(B)$ instead of $P(A) + P(B) - P(A\cap B)$.

(x) Several candidates tried incorrect methods of solution.

- Explain how to identify the regression lines as "y on x" and "x on y" separately, using the fact $b_{yx} \cdot b_{xy} = r^2$ and $r^2 < 1$. Also the fact that r, b_{xy}, b_{yx} are either all positive or all negative.
- Complex numbers need to be properly understood, both algebraically and graphically by the students. Modulus calculations and argument determination require thorough explanation.
- All the basic rules of probability need to be understood by students. Cases in favour can be found by counting correctly and the probability obtained as the rates of favourable cases to total number of cases. Alternatively the rule, $P(A\cup B) = P(A) + P(B) - P(A\cap B)$ could have been used if understood properly.
- All the various forms of differential equations and methods of solving them need thorough practice and revision.

MARKING SCHEME

Question 1.

(i)
$$A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} = \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix}$$

$$A^2 + xI_2 = yA$$

$$\Rightarrow \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix} + x\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = y\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} \qquad \boxed{\begin{matrix} y = 8 \\ x = 8 \end{matrix}}$$

(ii)
$$\frac{x^2}{9} - \frac{y^2}{25} = 1$$

$$a^2 = 9,\ b^2 = 25$$

$b^2 = a^2 (e^2-1)$

$$\Rightarrow \frac{25}{9} = e^2 - 1 \Rightarrow e^2 = \frac{34}{9}$$

$$e = \frac{\sqrt{34}}{3}$$

$$F(\pm\sqrt{34}, 0)$$

(iii) $$\tan\left[2\tan^{-1}\frac{1}{2} - \cot^{-1}\frac{1}{3}\right]$$

$$= \tan\left[\tan^{-1}\frac{1}{1-\frac{1}{4}} - \tan^{-1}\frac{1}{3}\right]$$

$$= \tan\left[\tan^{-1}\frac{4}{3} - \tan^{-1}\frac{1}{3}\right]$$

$$= \tan\left[\tan^{-1}\frac{\frac{4}{3}-\frac{1}{3}}{1+\frac{4}{3}\times\frac{1}{3}}\right]$$

$$= \frac{1}{\frac{13}{9}} = \frac{9}{13}$$

(iv) Let $A = \underset{x \to 0}{\text{Lt}} (1 + \sin x)^{\cot x}$

$\log A = \underset{x \to 0}{\text{Lt}} \cot x \log(1 + \sin x)$

$$= \underset{x \to 0}{\text{Lt}} \frac{\log(1+\sin x)}{\tan x}$$

$$= \underset{x \to 0}{\text{Lt}} \left(\frac{\frac{\cos x}{1+\sin x}}{\sec^2 x}\right) = 1$$

$= A = e$ or $\log A = 1$

(v) $\int e^x(2+\sin 2x)\,dx / \cos^2 x$

$= \int e^x (2\sec^2 x + 2\tan x)\,dx$

$= 2\int e^x \sec^2 x\,dx + 2(\tan x\, e^x - \int \sec^2 x\, e^x\, dx)$

$= 2e^x \tan x + c$

(vi)

$$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx. \text{ ....eqn. (1)}$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}\,dx.$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx. \text{ ....eqn. (2)}$$

Adding eqn. (1) and eqn. (2)

$$2\,I = \int_0^{\frac{\pi}{2}} 1\,dx.$$

$$2\,I = |x|_0^{\pi/2} = \frac{\pi}{2}$$

$$\therefore\ I = \frac{\pi}{4}$$

(vii) Let the line of regression of x on y be $3x-2y=5$ and y on x be $x-4y=7$.

Writing the first equation in the form $x = \frac{2}{3}\,y + \frac{5}{3}$, we get $b_{xy} = \frac{2}{3}$.

Writing the second equation in the form $y = \frac{1}{4}\,x - \frac{7}{4}$., we get $b_{yx} = \frac{1}{4}$.

Now $r^2 = b_{yx} \times b_{xy} = \frac{1}{4}\times\frac{2}{3} = \frac{1}{6}$

$r = \frac{1}{\sqrt{6}}$.Since r, b_{yx} and b_{xy} all have same sign.

(viii) $\dfrac{\left(1+\sqrt{3}i\right)^2}{\sqrt{3}-i} = \dfrac{1-3+2\sqrt{3}i}{\sqrt{3}-i}$

$$= \frac{-2+2\sqrt{3}i}{\sqrt{3}-i} \times \frac{\sqrt{3}+i}{\sqrt{3}+i}$$

$$= \frac{-4\sqrt{3}+4i}{3+1} = -\sqrt{3}+i$$

Modulus = $\sqrt{3+1} = 2$

argument = $\tan^{-1}\left(\dfrac{-1}{\sqrt{3}}\right) = \pi - \dfrac{\pi}{6} = \dfrac{5\pi}{6}$

(ix) P(No of multiple by 3 or 4) = $\dfrac{6}{20} + \dfrac{5}{20} - \dfrac{1}{20}$

$$= \frac{10}{20} = \frac{1}{2}$$

(x) (x + 1) dy = 2xy dx

$$\int \frac{dy}{y} = \int \frac{2x}{x+1} dx$$

$$\Rightarrow \textit{log}\ y = 2\int \frac{x+1-1}{x+1} dx = 2\int \left(1 - \frac{1}{x+1}\right) dx$$

$$\Rightarrow \textit{log}\ y = 2\left[x - \log\left(x+1\right)\right] + \textit{log}\ c$$

$$\Rightarrow \textit{log}\ \frac{y(x+1)^2}{c} = 2x$$

$$\frac{y\left(x+1\right)^2}{c} = e^{2x}$$

$$y = \frac{c\ e^{2x}}{\left(x+1\right)^2}$$

Question 2

(a) Using properties of determinants, prove that: **[5]**

$$\begin{vmatrix} a^2+1 & ab & ac \\ ba & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = a^2 + b^2 + c^2 + 1$$

(b) Using matrix method, solve the following system of equation: **[5]**

$x - 2y = 10$, $2x + y + 3z = 8$ and $-2y + z = 7$

<u>Comments of Examiners</u>

(a) Determinant properties were not correctly implemented by a number of candidates. Some also expanded the determinant directly, thereby going against the directions. Rows and columns not correctly identified by several candidates. Some candidates used irrelevant properties.

(b) A number of candidates could not calculate |A| correctly. Some could not obtain the adjoint and the inverse correctly owing to faulty application of cross multiplication rule. For, AB=C some concluded $B=C.A^{-1}$ instead of $B=A^{-1}.C$. Some used Cramer's rule which was not asked.

<u>Suggestions for teachers</u>

- Students need to understand that determinant properties are to be used in order to simplify the given determinant. Determinant Properties and their effective utilisation need to be explained properly with numerous examples. Plenty of practice will give the students an idea of how to obtain two zeroes in a line for easiest simplification.
- Inverse of a square matrix needs to be taught and done step by step. Utilisation of the inverse to correctly find the unknown matrix needs to be understood. Sufficient practice is a must.

MARKING SCHEME

Question 2.

(a)

$$\begin{vmatrix} a^2+1 & ab & ac \\ ba & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$$

Taking a, b c common from R1, R2, R3

$$=abc\begin{vmatrix} a+1/a & b & c \\ a & b+1/b & c \\ a & b & c+1/c \end{vmatrix}$$

$R1 \to R1 - R2,\ R2 \to R2 - R3$

$$=abc\begin{vmatrix} 1/a & -1/b & 0 \\ 0 & 1/b & -1/c \\ a & b & c+1/c \end{vmatrix}$$

Taking 1/a, 1/b, 1/c common from C1, C2, C3 respectively.

$$= \frac{abc}{abc}\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ a^2 & b^2 & c^2+1 \end{vmatrix}$$

$= (c^2+1+b^2) + a^2$

$= a^2+b^2+c^2 + 1$

(b)

$$A = \begin{pmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{vmatrix}$$

$= 11$

$$A^{-1} = 1/11\begin{pmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{pmatrix}$$

$X = A^{-1}B$

$$X = 1/11\begin{pmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{pmatrix}\begin{pmatrix} 10 \\ 8 \\ 7 \end{pmatrix}$$

$$= 1/11\begin{pmatrix} 44 \\ -33 \\ 11 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix} \quad \therefore\ x = 4,\ y = -3,\ z = 1$$

Question 3

(a) If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi,$ prove that: $x^2+y^2+z^2+2xyz=1$ **[5]**

(b) P, Q and R represent switches in on position and P′, Q′ and R′ represent switches in off position. Construct a switching circuit representing the polynomial PR + Q(Q′ + R)(P + QR). Using Boolean Algebra, simplify the polynomial expression and construct the simplified circuit. **[5]**

Comments of Examiners

(a) Some candidates got the formula, $\cos^{-1}x+\cos^{-1}y = \cos^{-1}\left(xy-\sqrt{1-x^2}\ \sqrt{1-y^2}\right)$ wrong. Also, some simplified $\cos\left(\pi-\cos^{-1}Z\right)$ as ‘Z’ instead of ‘–Z’. A few candidates changed $\cos^{-1}$ to $\sin^{-1}$ or $\tan^{-1}$ forms and could not simplify the cumbersome terms. Rational and irrational terms were not separated before squaring. Squaring and simplification errors were also observed.

(b) A few sketching errors were made but mostly simplification errors occurred while expanding the Boolean function. In some cases, the final simplified answer was given as RP+RQ instead of R (P+Q). The final diagram of the circuit thus was to have three switches instead of four.

Suggestions for teachers

- Laws of trigonometry need to be revised before learning inverse trigonometry functions. All algebraic and trigonometric laws need to be practiced sufficiently.
- Students must be taught that a.b and a+b depict series and parallel circuitry respectively. All Boolean algebra laws need to be well understood before application. Errors like PR+Q$\left(Q'+R\right)\left(P+QR\right)$ taken as $\left(PR+Q\right)\left(Q'+R\right)\left(P+QR\right)$, $Q.Q'=1$ and 1+Q=1 should not happen.

MARKING SCHEME

Question 3.

(a)

$$cos^{-1}x+cos^{-1}y+cos^{-1}z=\pi$$

$$\Rightarrow cos^{-1}x+cos^{-1}y=\pi-cos^{-1}z$$

$$\Rightarrow cos^{-1}\left(xy-\sqrt{1-x^2}\ \sqrt{1-y^2}\right)=\pi-cos^{-1}z$$

$$\Rightarrow xy-\sqrt{1-x^2}\ \sqrt{1-y^2}=cos\left(\pi-cos^{-1}z\right)$$

$$\Rightarrow xy-\sqrt{1-x^2}\ \sqrt{1-y^2}=-cos\ cos^{-1}z=-z$$

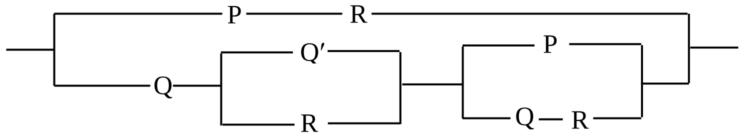
$$\Rightarrow xy+z=\sqrt{1-x^2}\ \sqrt{1-y^2}$$

squaring both sides

$$\cancel{x^2y^2}+z^2+2xyz=1-x^2-y^2+\cancel{x^2y^2}$$

$$\Rightarrow \boxed{x^2+y^2+z^2+2xyz=1}$$

(b)



= PR + Q(Q' + R) (P+ QR)

= PR + (QQ' + QR)(P + QR)

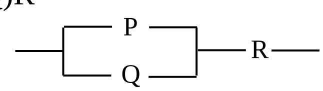
= PR + QR(P + QR) (QQ'=0)

= PR+PQR+ QR 1 + Q = 1

= PR(1+Q) + QR

= PR+QR = (P+Q)R

(simplified circuit)



Question 4

(a) Verify Rolle's Theorem for the function $f(x) = e^x(\sin x - \cos x)$ on $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$. **[5]**

(b) Find the equation of the parabola with latus-rectum joining points (4, 6) and (4, –2) **[5]**

Comments of Examiners

(a) Several candidates failed to state all the criteria for application of Rolle's Mean Value Theorem correctly. Some failed to differentiate the function correctly. The concept of closed and open interval was not clear to many.

(b) Candidates scored marks in this part of the question.

Suggestions for teachers

– Teachers need to help students enumerate the criteria for Mean Value Theorems correctly. Firstly, functions have to be continuous in the closed interval; secondly, the derivative needs to exist in the open interval and thirdly, f(a)=f(b) needs to be established. Copious practice is a must to understand derivative operations and that while solving $f'(x) = 0$ the value of x (say c) needs to be found in the open interval.

– Conics need to be understood well by noting details with regard to their sketching and derivation of their equations for standard forms as well as for other modified forms.

MARKING SCHEME

Question 4.

(a)

f(x) = e^x (sin x-cos x) is continuous in $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$

f(x) is differentiable in $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

f′(x) = e^x(cos x + sin x) + e^x(sin x – cos x)

= $2e^x$sin x

f′(x) exists in $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

f(π /4) = (5 π /4) = 0

there exist ‘c’ in $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ such that f ′(c) = 0

$2e^c$ sin c = 0, c = 0, π ⇒ c = π lies between $\frac{\pi}{4}$ and $\frac{5\pi}{4}$. Hence, Rolle’s Theorem is verified.

Question 5

(a) If y = $\frac{x\sin^{-1}x}{\sqrt{1-x^2}}$, prove that: $(1-x^2)\frac{dy}{dx} = x + \frac{y}{x}$ **[5]**

(b) A wire of length 50 m is cut into two pieces. One piece of the wire is bent in the shape of a square and the other in the shape of a circle. What should be the length of each piece so that the combined area of the two is minimum? **[5]**

Comments of Examiners

(a) $\frac{u}{v}$ rule for differentiating the function was incorrectly done by a number of candidates. Some candidates erred while differentiating after cross-multiplication.

(b) The required sum of areas of the two parts had to be expressed correctly and in terms of a single variable. Some candidates failed in this. Differentiation, and simplification errors were also made by several candidates.

Suggestions for teachers

- Differentiation rules for different functions and forms need continuous revision and practice. Obtaining the result in the required form needs simplification, cross multiplication, substitution, etc. hence enough revision and practice is required.
- If $4x+2\pi r=50$ then $r=\frac{50-4x}{2\pi}$. Hence $A=x^2+\pi\left(\frac{50-4x}{2\pi}\right)^2$. In this manner 'A' can be expressed in terms of a single variable 'x'. Differentiation and solving follows. Also, second order derivative needs to be shown as negative for maximum, and positive for minimum. Teachers need to encourage students to practice these problems regularly.

MARKING SCHEME

Question 5.

(a)

$$\frac{dy}{dx}=\frac{\left(\sin^{-1}x+\frac{x}{\sqrt{1-x^2}}\right)\sqrt{1-x^2}-x\sin^{-1}x\times\frac{-\cancel{2}x}{\cancel{2}\sqrt{1-x^2}}}{1-x^2}$$

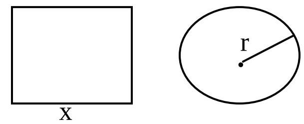
$$\left(1-x^2\right)\frac{dy}{dx}=\sqrt{1-x^2}\,\sin^{-1}x+x+xy$$

$$\left(1-x^2\right)\frac{dy}{dx}=\left(1-x^2\right)\frac{y}{x}+x+xy$$

$$\left(1-x^2\right)\frac{dy}{dx}=\frac{y}{x}-xy+x+xy$$

$$\left(1-x^2\right)\frac{dy}{dx}=x+\frac{y}{x}$$

(b) Let the side of the square be x and radius of the circle r.



$4x + 2\pi r = 50$

$2\pi r = 50 - 4x \Rightarrow r = \dfrac{50-4x}{2\pi}$

$A = x^2 + \pi r^2$

$= x^2 + \pi\left(\dfrac{50-4x}{2\pi}\right)^2 = x^2 + \dfrac{1}{4\pi}(50-4x)^2$

For $\min^m$ A, $\dfrac{dA}{dx} = 0$

$\dfrac{dA}{dx} = 2x + \dfrac{1}{4\pi}2(50-4x)(-4) = 0$

$\Rightarrow 2\pi x - 100 + 8x = 0$

$\Rightarrow x = \dfrac{50}{\pi+4}$

$\dfrac{d^2A}{dx^2} = +\text{ve}$, A is min^m at $x = \dfrac{50}{\pi+4}$

$\therefore$ length of square $= \dfrac{200}{\pi+4}$

Length of circle = $50 - \dfrac{200}{\pi+4} = \dfrac{50\pi}{\pi+4}$

Question 6

(a) Evaluate: $\displaystyle\int \frac{x+\sin x}{1+\cos x}\,dx.$ **[5]**

(b) Sketch the graphs of the curves $y^2 = x$ and $y^2 = 4 - 3x$ and find the area enclosed between them. **[5]**

Comments of Examiners

(a) After breaking up the integral correctly into two parts, some candidates incorrectly used Integration by parts rule. Conversion of sin x as $2\sin\frac{x}{2}\cos\frac{x}{2}$ and 1+cos x as $2\cos^2\frac{x}{2}$ was not remembered or utilised by some. Several candidates committed the mistake of ignoring the constant of integration.

(b) Sketching of both parabolas in the same diagram was not done by many candidates. Their points of integration were not found by some. The area required was not correctly identified in several cases. Some candidates expressed the definite integrals and their limits incorrectly.

Suggestions for teachers

- By-parts rule needs to be understood and applied correctly. Insufficient practice of such functions leads to errors.
- The parabolas, $y^2=x$ and $y^2=4-3x$ required accurate sketching. This needs to be taught well. Their points of intersection need to be calculated by solving the equations simultaneously.

MARKING SCHEME

Question 6.

(a)
$$I=\int\frac{x+\sin x}{1+\cos x}dx$$

$$=\int\frac{x+\sin x}{2\cos^2 x/2}dx$$

$$=\int\left(\frac{x}{2\cos^2 x/2}+\frac{2\sin x/2\cos x/2}{2\cos^2 x/2}\right)dx$$

$$=\int\left(\frac{1}{2}x\sec^2 x/2+\tan x/2\right)dx$$

$$=\frac{1}{2}\int x\sec^2\frac{x}{2}dx+\int\tan\frac{x}{2}dx$$

$$=\frac{1}{2}\left[x\frac{\tan x/2}{1/2}-\int\left(\frac{\tan x/2}{1/2}\right).1dx\right]+\int\tan\frac{x}{2}dx+c$$

$$=x\tan\frac{x}{2}-\int\tan\frac{x}{2}dx+\int\tan\frac{x}{2}dx+c$$

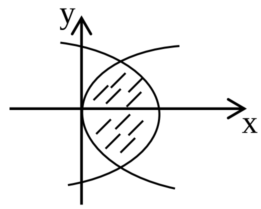
$$=x\tan\frac{x}{2}+c$$

(b) $y^2 = 4 - 3x$

$y^2 = x$

Solving (1) & (2) we get $x = 4 - 3x$

$4x = 4$, $x = 1$



$$\text{Required Area } = 2\left[\int_0^1 \sqrt{x}\ dx + \int_1^{4/3} \sqrt{4-3x}\ dx\right]$$

$= \quad 2\,(2/3 + 2/9)$

$= \quad 16/9$ sq units

Question 7

(a) A psychologist selected a random sample of 22 students. He grouped them in 11 pairs so that the students in each pair have nearly equal scores in an intelligence test. In each pair, one student was taught by method A and the other by method B and examined after the course. The marks obtained by them after the course are as follows: **[5]**

Pairs	1	2	3	4	5	6	7	8	9	10	11
Method A	24	29	19	14	30	19	27	30	20	28	11
Method B	37	35	16	26	23	27	19	20	16	11	21

Calculate Spearman's Rank correlation.

(b) The coefficient of correlation between the values denoted by X and Y is 0·5. The mean of X is 3 and that of Y is 5. Their standard deviations are 5 and 4 respectively. Find: **[5]**

(i) the two lines of regression

(ii) the expected value of Y, when X is given 14

(iii) the expected value of X, when Y is given 9.

<u>Comments of Examiners</u>

(a) Some candidates calculated the ranks incorrectly. Correction for $\sum d^2$ was either wrong or / and applied incorrectly in the formula for r. Several candidates even used Karl-Pearson's method for finding r.

(b) Many candidates were confused between the two regression equations. Some found y from given x by using the regression equation of x on y instead of the one of y on x.

<u>Suggestions for teachers</u>

– The formula needs to be correctly understood by students. The correction $\sum \frac{(m^3 - m)}{12}$ has to be added to $\sum d^2$ before use in the formula $1 - \frac{6\sum d^2}{n(n^2-1)}$. Sufficient practice can help students make a distinction between the two methods of finding r.

- Make students understand that $b_{yx} = r.\frac{\sigma y}{\sigma x}$ and $b_{xy} = r.\frac{\sigma x}{\sigma y}$. To find y from x students need to use $y-\overline{y}=b_{yx}(x-\overline{x})$. Students must be asked to take care and substitute the correct values of the terms in the two regression equations.

MARKING SCHEME

Question 7.

(a)

Pairs	A	B	Rank A	Rank B	D	D^2
1	24	37	6	1	5	25
2	29	35	3	2	1	1
3	19	16	8.5	9.5	1	1
4	14	26	10	4	6	36
5	30	23	1.5	5	-3.5	12.25
6	19	27	8.5	3	5.5	30.25
7	27	19	5	8	-3	9
8	30	20	1.5	7	-5.5	30.25
9	20	16	7	9.5	-2.5	6.25
10	28	11	4	11	-7	49
11	11	21	11	6	5	25
						$\sum d^2 = 225$

$$r = 1 - \frac{6\left[\sum d^2 + \sum \frac{1}{12}\left(m^3 - m\right)\right]}{n\left(n^2 - 1\right)}$$

n = no. of pair of observations

m = denotes, no. of times one observation is repeating

$= 1 - 6[225+(2^3-2)/12+(2^3-2)/12+(2^3-2)/12]/(11\times120)$

$= 1 - 6(225+1.5)/11\times120$

$= 1 - 1350/1320$

$= 1 - 1.029$

= - 0.029, shows a negative low correlation.

(b) $\overline{X} = 3,\ \overline{Y} = 5,\ \sigma_X = 5,\ \sigma_Y = 4,\ r = 0.5$

The line of regression of Y on X is given by $y - \overline{Y} = r\frac{\sigma_Y}{\sigma_X}(x - \overline{X})$.

$y - 5 = 0.5\times\frac{4}{5}(x-3)$. *i.e.* $5y - 25 = 2x - 6$

$2x - 5y + 19 = 0$ …eqn. (i)

The line of regression of X on Y is given by $x - \overline{X} = r\frac{\sigma_X}{\sigma_Y}(y - \overline{Y})$.

$x - 3 = 0.5\times\frac{5}{4}(y-5)$. *i.e.* $8x - 24 = 5y - 25$

$8x - 5y + 1 = 0$ …eqn. (2)

(i) Expected value of y when x = 14

$2\times14 - 5y + 19 = 0 \Rightarrow y = 9.4$

(ii) Expected value of x when y = 9

$8x - 5y + 1 = 0 \Rightarrow x = 5.5$

Question 8

(a) In a college, 70% students pass in Physics, 75% pass in Mathematics and 10% students fail in both. One student is chosen at random. What is the probability that: **[5]**

(i) He passes in Physics and Mathematics.

(ii) He passes in Mathematics given that he passes in Physics.

(iii) He passes in Physics given that he passes in Mathematics.

(b) A bag contains 5 white and 4 black balls and another bag contains 7 white and 9 black balls. A ball is drawn from the first bag and two balls drawn from the second bag. What is the probability of drawing one white and two black balls? **[5]**

Comments of Examiners

(a) Many candidates erred while finding $A \cap B$ and $A \cup B$. Errors were also made in using conditional probability formulae.

(b) The cases for withdrawal of balls from the bags in different colour combinations were not correctly identified by candidates. It was noticed that summation and product laws of probability were not understood by many candidates.

Suggestions for teachers

- Probability theory needs to be understood through copious practice. Independent cases or mutually exclusive situations need plenty of illustration. Conditional probability problems need to be taught with care.
- Separate the problems into cases and explain them to the students. Summation and product laws should be taught well from simple to difficult cases.

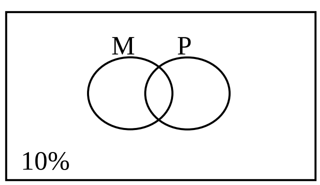
MARKING SCHEME

Question 8.

(a) $P(M) = 0.75$

$P(P) = 0.70$

$P(M \cup P) = 0.90$



(i) $P(M \cup P) = P(M) + P(P) - P(M \cap P)$

$0.90 = 0.75 + 0.70 - P(M \cap P)$

$\Rightarrow P(M \cap P) = 0.55$

(ii) $P(M/P) = \dfrac{P(M \cap P)}{P(P)} = \dfrac{0.55}{0.70} = \dfrac{11}{14}$

(iii) $P(P/M) = \dfrac{P(M \cap P)}{P(M)} = \dfrac{0.55}{0.75} = \dfrac{11}{15}$

(b) Bag 1 – 5W 4B, Bag 2 – 7 W 9 B

$$P(E) = \frac{5}{9} \times \frac{9c_2}{16c_2} + \frac{4}{9} \times \frac{7c_1 \times 9c_1}{16c_2}$$

$$\frac{\cancel{5}}{\cancel{9}} \times \frac{\cancel{9}}{2 \times \underset{3}{\cancel{15}}} + \frac{\cancel{4}}{\cancel{9}} \times \frac{7 \times \cancel{9}}{\underset{2}{\cancel{8}} \times 15}$$

$$= \frac{1}{6} + \frac{7}{30} = \frac{12}{30} = \frac{2}{5}$$

Question 9

(a) Using De Moivre's theorem, find the least positive integer n such that **[5]**

$\left(\dfrac{2i}{1+i}\right)^n$ is a positive integer.

(b) Solve the following differential equation: **[5]**

$(3xy + y^2)dx + (x^2 + xy)dy = 0$

<u>Comments of Examiners</u>

(a) Some candidates failed to convert the given complex number into correct polar form $r(\cos\theta + i\sin\theta)$ and correctly identify r and θ. De Moivre's s theorem was correctly applied by most candidates but some did not note that a positive integral value of the function was required for the least positive integral value of n. Some chose value of n as 0 which is not a positive integer. A few chose n=4 but that made $\cos\dfrac{n\pi}{4}$ a negative integer.

(b) Many candidates proceeded to solve by correctly using the rules for homogeneous equations but after putting $y = vx$ erred in noting that $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$ or erred while simplifying and separating the v and x functions. The subsequent integrals were not correctly understood by some.

<u>*Suggestions for teachers*</u>

- Firstly make the students understand that $\dfrac{2i}{1+i} = 1 + i = \sqrt{2}\left(\cos\dfrac{\pi}{4} + i\sin\dfrac{\pi}{4}\right)$ in polar form. $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ etc. need to be clarified by the teachers for better understanding of the students. Basics of algebra and trigonometry must be revised sufficiently.
- Homogeneous differential equations and their solution needs careful explanation and demonstration by teachers. All forms of integration need extra practice and drill. Tell students that the constant of integration must not be ignored.

MARKING SCHEME

Question 9.

(a) We have $\frac{2i}{1+i} = \frac{2i}{1+i} \times \frac{1-i}{1-i} = \frac{2(i+1)}{2} = 1+i$

$$= \sqrt{2}\left[\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right]$$

$$= \sqrt{2}\left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right]$$

$$\therefore \left(\frac{2i}{1+i}\right)^n = \left(\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right)^n = 2^{\frac{n}{2}}\left(\cos\frac{n\pi}{4} + i\sin\frac{n\pi}{4}\right)$$

Which is positive integer

If $\frac{n\pi}{4} = 0, 2\pi, 4\pi, 6\pi, \ldots.$

$\rightarrow n = 0, 8, 16, 24, \ldots$

$\rightarrow$ *the least value of* n *is* $8.$

(b) $$\frac{dy}{dx} = -\left(3xy + y^2\right) / \left(x^2 + xy\right)$$

Put $y = vx$

$v + x dv/dx = -(3v+v^2)/(1+v)$

$x\, dv/dx = 1-(3v+v^2)/(v+1) - v$

$$\int 2(v+1)dv / \left(v^2 + 2v\right) = -4\int dx / x$$

$$\log\left|v^2 + 2v\right| = -4\log x + \log c$$

$$\log\left|v^2 + 2v\right| = \log c/x^4$$

$\log(y^2+2xy)/x^2 = \log c/x^4$

$y^2+2xy = c/x^2$

SECTION B

Question 10

(a) In a triangle ABC, using vectors, prove that $c^2 = a^2 + b^2 - 2ab\ \cos C$. **[5]**

(b) Prove that: $\vec{a}\,.\left(\vec{b}+\vec{c}\right)\times\left(\vec{a}+2\vec{b}+3\vec{c}\right)=\left[\vec{a}\,\vec{b}\,\vec{c}\right]$ **[5]**

Comments of Examiners

(a) Vector symbols were not used by many candidates. Diagrams drawn by candidates did not show arrows to indicate directions of $\vec{a}$, $\vec{b}$, $\vec{c}$.

(b) Many candidates performed dot product first, followed by cross product. This was incorrect. They simplified and arrived at the result somehow but it was not driven by logic.

Suggestions for teachers

- Teachers need to take care while teaching vector topics. Vectors need to be correctly depicted. Dot and cross products need thorough attention.
- Dot product produces a scalar quantity and subsequent cross-product of scalar with vector is meaningless. Combination of dot and cross product in scalar triple product needs thorough understanding. Vector algebra in totality need to be understood by the students, specifically properties of Scalar Triple Product.

MARKING SCHEME

Question 10.

(a)

$\vec{a}+\vec{b}+\vec{c}=0$

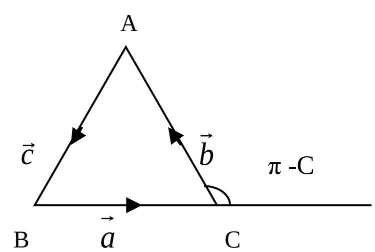
$\vec{a}+\vec{b}=-\ \vec{c}$

$\left(\vec{a}+\vec{b}\right)^2=\vec{c}^2$

$b^2+a^2+2\ \vec{a}.\vec{b}=c^2$

$a^2+b^2+2ab\ \cos(\pi-C)=c^2$

$a^2+b^2-2ab\cos C=c^2$



(b) LHS = $\vec{a}\,.\left(\vec{b}+\vec{c}\right)\times\left(\vec{a}+2\vec{b}+3\vec{c}\right)$

$= \vec{a}.\left[\vec{b}\times\vec{a}+\vec{b}\times 2\vec{b}+\vec{b}\times 3\vec{c}+\vec{c}\times\vec{a}+c\times 2\vec{b}+\vec{c}\times 3\vec{c}\right]$

$$= \vec{a}.\left[\vec{b}\times\vec{a}+3\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+2\ \vec{c}\times\vec{b}\right]$$

$$= \vec{a}.\left(\vec{b}\times\vec{a}\right)+\vec{a}.\left(3\vec{b}\times\vec{c}\right)+\vec{a}.\left(\vec{c}\times\vec{a}\right)+\vec{a}.\left(2\ \vec{c}\times\vec{b}\right)$$

$$= 0 + 3\left[\vec{a}\,\vec{b}\ \vec{c}\right]+0-2\left[\vec{a}\,\vec{b}\ \vec{c}\right]$$

$$= \left[\vec{a}\,\vec{b}\,\vec{c}\right]$$

Question 11

(a) Find the equation of a line passing through the points P (-1, 3, 2) and Q (-4, 2, -2). Also, if the point R (5, 5, λ) is collinear with the points P and Q, then find the value of λ. **[5]**

(b) Find the equation of the plane passing through the points (2, -3, 1) and (-1, 1, -7) and perpendicular to the plane $x - 2y + 5z + 1 = 0$. **[5]**

Comments of Examiners

(a) After expressing the equation of the line in Cartesian symmetric form, a few candidates failed to find λ by substituting coordinates of (5, 5, λ) in the equation of the line. Candidates who used vector form of the line also made a similar mistake.

(b) Some candidates wrote the equation of the Plane in Cartesian form but made mistakes while calculating a, b, c values by rule of cross-multiplication.

Suggestions for teachers

– Equation of a line in all forms needs to be taught well. Direction ratios of line and parallel perpendicular lines need clarity of understanding.

– Equation of line in symmetric and vector forms and their interchangeability needs proper explanation.

– Equation of a plane needs to be taught with patience and perseverance. Cartesian and vector forms need explanation.

MARKING SCHEME

Question 11.

(a) The equation of a line passing through two points P (– 1, 3, 2) and Q (–4, 2, – 2) is given by:

$$\frac{x+1}{-4+1}=\frac{y-3}{2-3}=\frac{z-2}{-2-2}$$

$$\frac{x+1}{-3}=\frac{y-3}{-1}=\frac{z-2}{-4}$$

Or

$$\frac{x+1}{3} = \frac{y-3}{1} = \frac{z-2}{4} \text{ ………eqn. (1)}$$

Now, if the three points P (– 1, 3, 2), Q (– 4, 2, – 2) and R (5, 5, λ) are collinear, then the coordinates of point R must satisfy the equation(1)

$$\frac{5+1}{3} = \frac{5-3}{1} = \frac{\lambda-2}{4}$$

$$\frac{6}{3} = \frac{2}{1} = \frac{\lambda-2}{4} \Rightarrow \lambda - 2 = 8 \ \textit{i.e.}\ \lambda = 10$$

(b) Equation of plane passing through (2, -3, 1)

a(x-2) + b(y+3)+c(z-1) = 0

a(-1 -2) + b(1+3) + c(-7 -1) = 0

-3a + 4b - 8c = 0

The given plane is perpendicular to x – 2y + 5z = 0

a – 2b + 5c = 0

Solving 1 and 2 we get a = 4k, b = 7k, c = 2k

4(x -2) + 7(y+3) + 2(z – 1) = 0

4x + 7y + 2z + 11 = 0

Question 12

(a) In a bolt factory, three machines A, B and C manufacture 25%, 35% and 40% of the total production respectively. Of their respective outputs, 5%, 4% and 2% are defective. A bolt is drawn at random from the total production and it is found to be defective. Find the probability that it was manufactured by machine C. **[5]**

(b) On dialling certain telephone numbers, assume that on an average, one telephone number out of five is busy. Ten telephone numbers are randomly selected and dialled. Find the probability that at least three of them will be busy. **[5]**

Comments of Examiners

(a) While using Baye's theorem, some candidates used actual numbers given instead of probabilities of the events. Probability of an event and the conditional probability for the happening of an event were not understood and also incorrectly substituted in several cases.

(b) Probability distribution theory was incorrectly applied by some candidates owing to carelessness or lack of understanding. In the expansion of $(q + p)^n$, some took the value of n as 3 instead of 10 by mistake.

Suggestions for teachers

– Baye's Theorem needs proper explanation and illustration. That the required probability is a ratio of conditional probability of a specific known event to the sum of the conditional probabilities of all events under consideration, needs to be taught so that on the part of students, the understanding is complete.

– This topic needs to be taught after a thorough revision of the binomial theorem. Meaning of each term in the expansion must be explained by teachers. Students must understand the meaning of "at least 3" and "at most 3" situations.

MARKING SCHEME

Question 12.

(a) P(A) = ¼, P(B) = 7/20, P(C) = 2/5

Let D be the probability of drawing a defective bolt.

P(D/A) = 1/20, P(D/B) = 1/25, P(D/C) = $\frac{1}{50}$

$$P(C/D) = \frac{P(C)\times P(D/C)}{P(A)P(D/A)+P(B)P(D/B)+P(C)P(D/C)}$$

$$= \frac{2/5\times 1/50}{(1/4\times 1/20+7/20\times 1/25+2/5\times 1/50)}$$

= 16/69

(b) $p = \frac{1}{5},\ q = \frac{4}{5},\ n = 10$

$P(x \geq 3) = 1 - P(x = 0_1\ 1\ 2)$

$$= 1 - \left[10c_0\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^{10} + 10c_1\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^9 + 10c_2\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^8\right]$$

$$= 1 - \left[\left(\frac{4}{5}\right)^{10} + 10 \times \frac{1}{5} \times \left(\frac{4}{5}\right)^{9} + 45 \times \frac{1}{25} \times \left(\frac{4}{5}\right)^{8} \right]$$

$$= 1 - \left(\frac{4}{5}\right)^{8} \left[\frac{16}{25} + 10 \times \frac{4}{25} + \frac{4}{25} \right]$$

$$= 1 - 0.678 = 0.322$$

SECTION C

Question 13

(a) A person borrows ₹ 68,962 on the condition that he will repay the money with compound interest at 5% per annum in 4 equal annual instalments, the first one being payable at the end of the first year. Find the value of each instalment. **[5]**

(b) A company manufactures two types of toys A and B. A toy of type A requires 5 minutes for cutting and 10 minutes for assembling. A toy of type B requires 8 minutes for cutting and 8 minutes for assembling. There are 3 hours available for cutting and 4 hours available for assembling the toys in a day. The profit is ₹ 50 each on a toy of Type A and ₹ 60 each on a toy of type B. How many toys of each type should the company manufacture in a day to maximize the profit? Use linear programming to find the solution. **[5]**

Comments of Examiners

(a) Instead of Present Value of an Annuity some candidates used Amount of an Annuity formula by mistake. Some could not calculate $(1 + i)^{-n}$ correctly, even with mathematical table or calculator.

(b) Most mistakes happened while expressing the constraints in the form of equations with both sides expressed in the same units. Some candidates omitted the graphical representation of the inequations and erred in finding the feasible area and points.

Suggestions for teachers

– Correct and meaningful understanding of the terms and situations involved is a must. Plenty of practice needs to be given to students.

– Linear programming as a topic requires knowledge of line sketching. Hence a little revision in this matter is essential. Correct depiction and shading of the required area is a big advantage. Neat diagrams help a lot. Casual approach is to be avoided and little extra practice is required.

MARKING SCHEME

Question 13.

(a) Let the installment be a.

The money borrowed is Rs.68,962, which is present worth of the annuity of a payable for 4 years (payable annually) at 5 % per annum.

$$\therefore\ P = Rs.68962,\ n = 4 \text{ and } i = \frac{5}{100} = \frac{1}{20} = \bullet 05$$

$$\text{Now,}\quad P = \frac{a}{i}\left\{1-(1+i)^{-n}\right\} \qquad 68962 = \frac{a}{\bullet 05}\left\{1-(1\bullet 05)^{-4}\right\}$$

Let $x = (1\bullet 05)^{-4}$

$$\therefore \log x = -4\log 1\bullet 05 = -4\times 0\bullet 0212$$

$$= -0\bullet 0848 = -1+1-0\bullet 0848$$

$$= \bar{1}\bullet 9152$$

$$\therefore\ x = \text{anti}\log \bar{1}\bullet 9152 = 0\bullet 8226$$

$$\therefore\ 68962 = \frac{a}{\bullet 05}\{1-0\bullet 8226\} = \frac{a}{\bullet 05}\{0\bullet 1774\}$$

$$a = \frac{68962\times\bullet 05}{0\bullet 1774} = 19436\bullet 87/19448.10$$

(Therefore, required amount of each installment = Rs.19436.87)

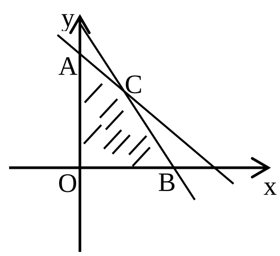
(b) Let x units of type A toys and y units of type B toys.

Maximize Z = 50x + 60 y

Subject to constrains

$$\left.\begin{aligned} 5x + 8y &\le 180 \\ 10x + 8y &\le 240 \end{aligned}\right\}$$

$x \ge 0,\ y \ge 0$



Solving graph, we get A(0, 22.5), B(24, 0), C(12, 15)

At A, z = 50× 0+60×22.5 = Rs. 1350

B, z = 50×24+60×0 = Rs.1200

C, z = 50×12+60×15 = Rs. 1500

Maximum profit is Rs.1500, when 12 units of type A and 15 units of type B is produced.

Question 14

(a) A firm has the cost function C = $\frac{x^3}{3} - 7x^2 + 111x + 50$ and demand function $x = 100 - p$. **[5]**

(i) Write the total revenue function in terms of x.

(ii) Formulate the total profit function P in terms of x.

(iii) Find the profit maximising level of output x.

(b) A bill of ₹ 5050 is drawn on 13th April 2013. It was discounted on 4th July 2013 at 5% per annum. If the banker's gain on the transaction is ₹ 0.50, find the nominal date of the maturity of the bill. **[5]**

Comments of Examiners

(a) Revenue and profit functions were incorrectly stated by some candidates. Profit function was not correctly differentiated by a few. The concept of the maxima and minima for profit maximisation was not known by a few candidates.

(b) A number of candidates used the wrong formula for bankers gain. Some found the Legal Due Date and did not work out the Nominal Due Date. Quadratic equation was incorrectly solved by some candidates.

Suggestions for teachers

– Explain terms like Cost and Demand functions. Revenue and profit functions too require revision.

– Bills of exchange should be explained thoroughly. Terms like Bankers discount, True discount and banker's gain need to be understood well. Their inter-relationship and formulae to find them need correct learning and recall.

MARKING SCHEME

Question 14.

(a)(i) Total revenue function = x(100 – x)

Profit function = Revenue function – Cost function

$= 100x - x^2 - (x^3/3 - 7x^2 + 111x+50)$

$= -x^3/3 + 6x^2 - 11x - 50$

$dP/dx = -x^2 + 12x - 11$

For maximum or minimum $dP/dx = 0$

$-x^2 + 12x + 11 = 0$

$x = 1, 11$

$d^2P/dx^2 = -2x + 12$

$= -2\times 11 + 12$

$= -10 < 0$

Profit is maximum when x = 11

(b) Let the unexpired period of bill at the time of discounting be t years.

$B.G = \frac{A(ni)^2}{1+ni}$, where A is the face value of the bill.

Here, $A = 5050, \quad i = 0.05$

$$0.50 = \frac{5050(0.05t)^2}{1+0.05t}$$

$$505t^2 - t - 20 = 0$$

$$\Rightarrow t = \frac{1 \pm \sqrt{1+40400}}{1010}$$

$$\Rightarrow t = \frac{1 \pm 201}{1010}$$

$$\Rightarrow t = \frac{1}{5} \qquad \text{(neglecting –ve value)}$$

$$\Rightarrow t = \frac{1}{5} = 73\ days$$

Thus, legal due date of maturity is 73 days after 4th July, which comes to 15 September.

$27\ days\ in\ July + 31\ days\ in\ August + 15\ days\ in\ Sept$

Therefore, the legal due date is 15th September and the nominal due date is 12th September.

Question 15

(a) The price of six different commodities for years 2009 and year 2011 are as follows: **[5]**

Commodities	A	B	C	D	E	F
Price in 2009 (₹)	35	80	25	30	80	x
Price in 2011 (₹)	50	y	45	70	120	105

The Index number for the year 2011 taking 2009 as the base year for the above data was calculated to be 125. Find the values of x and y if the total price in 2009 is ₹ 360.

(b) The number of road accidents in the city due to rash driving, over a period of 3 years, is given in the following table: **[5]**

Year	Jan - Mar	April - June	July-Sept.	Oct – Dec.
2010	70	60	45	72
2011	79	56	46	84
2012	90	64	45	82

Calculate four quarterly moving averages and illustrate them and original figures on one graph using the same axes for both.

Comments of Examiners

(a) Instead of the Simple Aggregate method, some incorrect methods were used by candidates. The value of y was not found by a few candidates.

(b) The averages were not correctly calculated by some candidates. Some erred while finding centred moving averages. Plotting of the centred averages was inaccurate in many cases.

Suggestion for teachers

– Calculation of Index Number by various methods requires sufficient practice for full comprehension.

MARKING SCHEME

Question 15.

(a)

Commodities	Price in 2009 (in R)	Price in 2011 (in R)
A	35	50
B	80	y
C	25	45
D	30	70
E	80	120
F	x	105
	$\Sigma P_0 = 250 + x$	$\Sigma P_1 = 390 + y$

Given, $\Sigma P_0 = 360$

$\Rightarrow 250 + x = 360 \Rightarrow x = 110$

Price index = $\frac{\Sigma P_1}{\Sigma P_0} \times 100$

$\Rightarrow 125 = \frac{390 + y}{360} \times 100$

4500 = 3900 + 10 y

Therefore, y = 60

(b) Calculations for trend by four quarterly moving averages:

Year	Quarter	Values	4-quarterly moving total	Four quarterly moving average	Four quarterly moving average centered
2010	I	70			
	II	60			
			247	247/4 = 61.75	
	III	45			125.75/2 = 62.875
			256	256/4 = 64.00	
	IV	72			127/2 = 63.500
			252	252/4 =63.00	
2011	I	79			126.25/2 = 63.125
			253	253/4 =63.25	
	II	56			129.50/2 = 64.750
			265	265/4 =66.25	
	III	46			135.25/2 = 67.625
			276	276/4 =69.00	

	IV	84			140/2 = 70.000
			284	284/4 =71.00	
2012	I	90			141.75/2 = 70.875
			283	286/4 =70.75	
	II	64			141/2 = 70.500
			281	281/4 =70.25	
	III	45			
	IV	82			

Correct Graph to be drawn.

Note: For questions having more than one correct solution, alternate correct solutions, apart from those given in the marking scheme, have also been accepted.

GENERAL COMMENTS:

(a) Topics found difficult by candidates in the Question Paper:

- Application of L' Hospitals rule for indeterminate forms.
- Determinant properties and their use.
- Indefinite integrals of all forms.
- Definite Integrals and area finding.
- Differentiation and applications.
- Inverse trigonometric functions.
- Conics in general.
- Probability theory.
- Vectors and vector applications.
- Probability distribution theory.
- Plane and straight line in 3- dimensions.

(b) Concepts in which candidates got confused:

- Solving equations by matrix and determinant methods.
- Trigonometric functions and Inverse trigonometric functions.
- Regression line of x on y and yon x.
- Parallel and series circuitry of Boolean functions.
- Open and closed intervals for Mean Value Theorems.
- Scalar and Vector product of Vectors.
- Annuity due and immediate Annuity.
- Demand function and Revenue function.
- Aggregate and Price-Relative methods for finding Index Numbers.

(c) Suggestions for students:

- Learn and absorb finer details of a topic.
- Cultivate a questioning attitude.
- Attempt the question with a tension-free approach.
- Make wise choices from the options available.
- Learn to use the correct formula for solving a problem.
- Read the question carefully and answer accordingly.
- All steps of calculations need to be simplified before proceeding to the next step.
- Revise and practice from previous years question papers and sample papers.